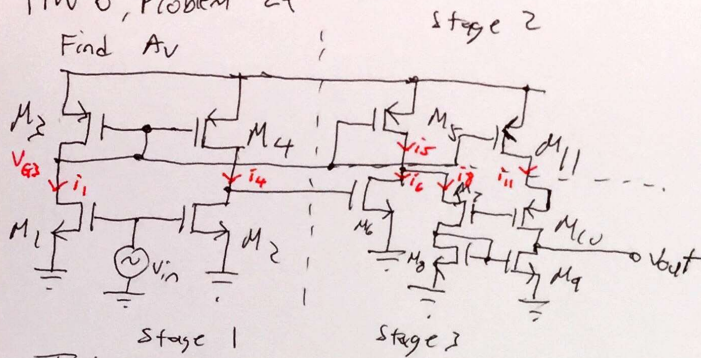


HW 6, Problem 2f

Find A_v

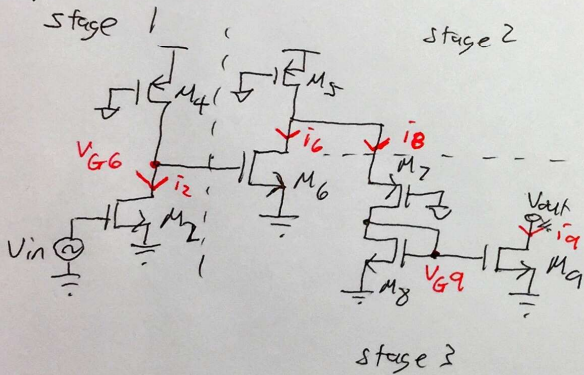


$$R_{out} = (r_{dsq}) \parallel (r_{ds10} + r_{ds11} + g_{m10} r_{ds10})$$

Idea: We know $G_M = \frac{i_{out}}{v_{in}}$, so we want to find the current from M_1/M_2 to the output node. We will use superposition to find the contributions from M_1 and M_2 separately.

Part I: M_1 off, M_2 on (superposition)

Simplified circuit:



Since M_1 is off, $I_1 = 0$, gate voltage for $M_2/M_4/M_5/M_{11} = 0$

Stage 1: $i_2 = g_{m2} \cdot v_{in}$
 $r_{V_{GG}} = r_{ds4} \parallel r_{ds2}$
 $V_{GG} = i_2 \cdot r_{V_{GG}} = (g_{m2})(r_{ds4} \parallel r_{ds2}) v_{in}$

Stage 2: $i_6 = g_{m6} \cdot V_{GG}$
 $i_8 = -i_6$

Stage 3: $V_{G9} = \frac{1}{g_{m8}} \cdot i_8$

$i_9 = g_{m9} \cdot V_{G9} = -\frac{g_{m9}}{g_{m8}} i_6 = \left[-\left(\frac{g_{m9}}{g_{m8}}\right)(g_{m6})(g_{m2})(r_{ds4} \parallel r_{ds2}) v_{in} \right]$

$i_{M2} \downarrow$

Part II: M_1 on, M_2 off (superposition)

Find $V_{G3} = V_{G4} = V_{G5} = V_{G11}$

$$i_1 = g_{m1} V_{in}$$

$$V_{G3} = g_{m1} V_{in} \left(\frac{1}{g_{m3}} \parallel r_{ds1} \right) \approx \frac{g_{m1}}{g_{m3}} V_{in}$$

a. Find current contribution due to M_4

$$i_4 = \frac{g_{m4}}{g_{m3}} \cdot i_1 \quad \leftarrow \text{stage 1}$$

$$V_{G6} = +i_4 \cdot (r_{ds4} \parallel r_{ds2}) \quad \left. \vphantom{V_{G6}} \right\} \text{stage 3}$$

$$i_6 = g_{m6} \cdot V_{G6}$$

$$i_8 = -i_6, \quad V_{G9} = \frac{1}{g_{m8}} \cdot i_8$$

$$i_9 = \frac{g_{m9}}{g_{m8}} \cdot i_8 = \boxed{-\frac{g_{m9}}{g_{m8}} \cdot g_{m6} \cdot \frac{g_{m4}}{g_{m3}} \cdot (r_{ds4} \parallel r_{ds2}) \cdot g_{m1} \cdot V_{in}}$$

$I_{M1,A}$
↓

b. Find current contribution due to M_5

$$i_5 = V_{G3} g_{m5}$$

$$i_8 = i_5$$

$$V_{G9} = \frac{1}{g_{m8}} i_8$$

$$i_9 = \frac{g_{m9}}{g_{m8}} i_8 = \boxed{\frac{g_{m9}}{g_{m8}} \cdot g_{m5} \cdot \frac{g_{m1}}{g_{m3}} V_{in}}$$

$I_{M1,B}$
↓

c. Find current contribution due to M_{11}

$$i_{11} = V_{G3} g_{m11} = \boxed{-g_{m11} \cdot \frac{g_{m1}}{g_{m3}} V_{in}} \quad \leftarrow I_{M1,C}$$

$$G_M = \frac{V_{out}}{V_{in}} = \boxed{I_{M2} + I_{M1,A} + I_{M1,B} + I_{M1,C}}$$