HKN ECE 329 Exam 1 Review Session

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Topics

- Vector Calculus Review
- Coulomb's Law, Lorentz Force
- Gauss' Law
- Boundary Conditions
- Maxwell's Equations and Continuity Equation
- Conductors and Dielectrics
  - Polarization
- Capacitance and Conductivity

Vector Calculus Review

- Dot Product: $< x_a, x_b, x_c > \cdot < y_a, y_b, y_c > = x_a y_a + x_b y_b + x_c y_c \ - Scalar!$
- Cross Product: $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{x}(A_y B_z - A_z B_y) - \hat{y}(A_x B_z - A_z B_x) + \hat{z}(A_x B_y - A_y B_x) - Vector!$
- Curl: $\nabla \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \hat{x}\left(\frac{\partial}{\partial y} A_z - \frac{\partial}{\partial z} A_y\right) - \hat{y}\left(\frac{\partial}{\partial x} A_z - \frac{\partial}{\partial z} A_x\right) + \hat{z}\left(\frac{\partial}{\partial x} A_y - \frac{\partial}{\partial y} A_x\right) - Vector!$
- Divergence: $\nabla \cdot \vec{A} = < \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} > \cdot < A_x, A_y, A_z >$
- Gradient: $\nabla \mathrm{A} = < \frac{\partial A}{\partial x}, \frac{\partial A}{\partial y}, \frac{\partial A}{\partial z} >$

Vector Calculus Review

- Fundamental Theorem of MV Calculus: Useful for evaluating conservative fields

$$\int_a^b \nabla f \cdot d\vec{l} = f(b) - f(a), f \text{ } is \text{ } a \text{ } conservative \text{ } field$$

- Stokes' Theorem: What happens if E is a conservative field?

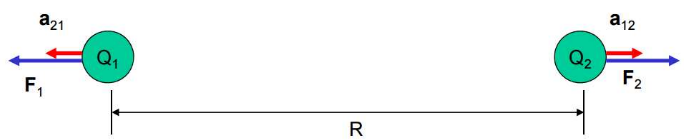
$$\oint \vec{E} \cdot d\vec{l} = \iint \left(\nabla \times \vec{E}\right) \cdot d\vec{S}$$

- Divergence Theorem:

$$\iiint (\nabla \cdot \vec{x}) dV = \iint \vec{x} \cdot d\vec{S}$$

Coulomb's Law and Lorentz Force

- Coulomb's Law: relates the forces between two charges



$$\vec{F}_1 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\hat{a}_{21} \qquad \vec{F}_2 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\hat{a}_{12}$$

- Electric field is the force per unit charge, units of Newtons/coulomb or Volts/meter
- Apply superposition to find the sum the force or E-field contributions from multiple charges
- Lorentz Force: $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$, relates forces on charged particle due to external electric and magnetic fields.

Gauss’ Law

- Gauss’ Law for Electric Fields tells us the electric flux due to an enclosed charge

$$\oiint \vec{D} \cdot d\vec{S} = Q_{enclosed}$$

$$\nabla \cdot \vec{D} = \rho$$

- We can also use Gauss’ Law to determine the E-field at a surface due to a given charge distribution. We must choose a convenient symmetry such that we know that the E-field is uniform on the surface
    - Ex: cylinder to enclose a line of charge
- We also have Gauss’ Law for Magnetic Fields:

$$\oiint \vec{B} \cdot d\vec{S} = 0$$

Boundary Conditions

- These are a big deal!

- $E_{1t} = E_{2t}$
  - Tangential electric fields on either side of the boundary must be equal

- $D_{1n} - D_{2n} = \rho$
  - The difference between two incident normal electric flux densities is the surface charge density

Maxwell's Equations and Continuity Equation

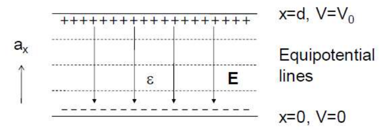
Faraday's Law	$\oint_C \vec{E} \bullet d\vec{l} = -\frac{d}{dt}\iint_S \vec{B} \bullet d\vec{S}$	$\nabla \times \vec{E} = -\frac{d\vec{B}}{dt}$
Ampere's Law	$\oint_C \vec{H} \bullet d\vec{l} = \iint_S \vec{J} \bullet d\vec{S} + \frac{d}{dt}\iint_S \vec{D} \bullet d\vec{S}$	$\nabla \times \vec{H} = \vec{J} + \frac{d\vec{D}}{dt}$
Gauss' Law	$\oiint_S \vec{B} \bullet d\vec{S} = 0$	$\nabla \bullet \vec{B} = 0$
Gauss' Law	$\oiint_S \vec{D} \bullet d\vec{S} = \iiint_V \rho dV$	$\nabla \bullet \vec{D} = \rho$
Continuity Eq.	$\oiint_S \vec{J} \bullet d\vec{S} = -\frac{d}{dt}\iiint_V \rho dV$	$\nabla \bullet \vec{J} = -\frac{d\rho}{dt}$

Conductors and Dielectrics

- Conductors:
  - E-field is always zero inside!
  - Image charges will form in the presence of an external E-field in order to guarantee the E-field is zero inside
  - $\vec{J} = \sigma\vec{E}$
- Dielectrics:
  - Atoms can form dipoles in the presence of an electric field where the nucleus and electrons become polarized
  - Polarization Vector: $\vec{P} = \epsilon_0 \chi_e \vec{E}_{tot}$
  - Connecting $\vec{D}, \vec{E}, and\ \vec{P}$: $\vec{D} = \vec{P} + \epsilon_0 \vec{E}_{total} = \vec{E}_{total}\epsilon_0(1+\chi_e), 1+\chi_e = dielectric\ constant$

Capacitance and Conductivity

- $C = \frac{Q}{V} = \frac{\epsilon A}{d}$
- Steps to find Capacitance:



- Laplace Equation $\vec{\nabla}^2 V = 0$
- Find V using boundary conditions
- Find **E** using $\vec{E} = -\vec{\nabla} V$
- Find **D** using $\vec{D} = \varepsilon \vec{E}$
- Get surface charge density on <u>one</u> conductor using BC $\rho_s = \vec{a}_n \bullet (D_{n1} - D_{n2})$
- Charge $Q = (Area)(\rho_s)$
- Capacitance $C = Q / V_0$

$V(x) = V_0 \frac{x}{d}$

$\rho = \varepsilon V_0 / d$

$C = \frac{\varepsilon A}{d}$ 3

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- AC Conductivity: $\sigma = \frac{Nq^2}{m(j\omega + \overline{\omega})}$
- DC Susceptibility: $\chi_e = \frac{N_d q^2}{m\omega_0^2 \epsilon_0}$